1. Introduction

This NMRA Data Sheet describes easements that smoothly connect circular curves to tangent (straight) track. It also provides guidance on how to lay out easements while planning and constructing a model railroad.

2. Background

Curved track is almost always unavoidable. Prototype railroads prefer tangent track but must use curves to minimize grades and to get around natural or manmade obstacles.

A train moving at mainline speeds on a circular curve experiences a lateral centrifugal force. When a train enters a circular curve directly from a tangent, it experiences a sudden lateral jerk due to the sudden increase in centrifugal force. In addition to its more popular meaning, jerk is a valid engineering term defined as the rate of change of acceleration.

In the earliest days of railroading, when speeds were comparatively slow, jerk was simply accepted. As train speeds increased, jerk increased intolerably. To minimize jerk, it became common practice to use an easement track (a.k.a. a transition curve) of gradually decreasing radius, inserted between the tangent and the circular curve. An easement gently increases, or *eases*, the centrifugal force from zero on the tangent to its full amount on the circular curve.

At trackside, the visual difference between an easement and a circular curve is difficult to see. A higher vantage point is necessary. Figure 1 is a satellite photo from Google Maps that shows a "top view" of a Norfolk Southern curve near Elkton, Virginia. Its easements are still difficult to see in the photo until the colored lines are overlaid. The straight blue lines lie on

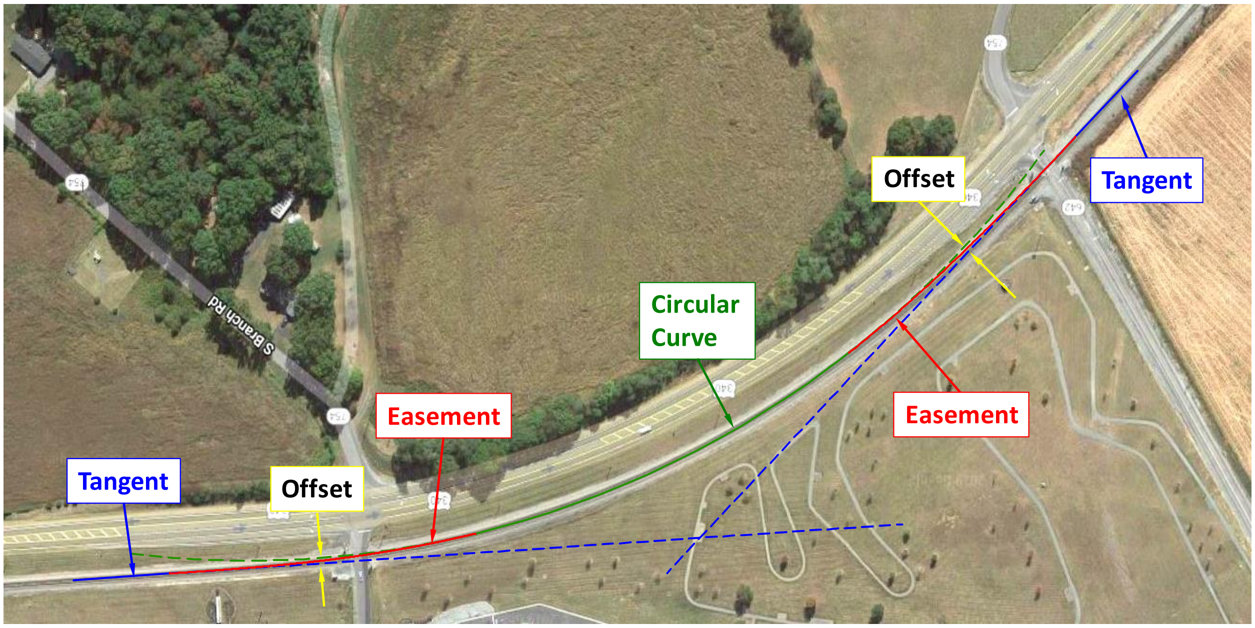

Figure 1: Prototype Curve with Easements (Imagery © 2019 Google Map data near Elkton, Virginia, USA, 38.381023N 78.651675W. Image rotated 180 degrees.

the tangents at each end, while the curved green line lies on the circular curve. The small offset, or gap, between them indicates the presence of easements, shown by the red lines. The dashed blue and green lines are extensions that help visualize the offset.

Prototype and model railroads both benefit from easements, albeit for different reasons. Simply stated, prototype railroads use easements to manage lateral forces. Model railroads, with their inherently sharper curves, use easements to minimize coupler swing, especially between cars of different lengths. Excessive swing can cause coupler and/or diaphragm binding and possible derailment. Using an easement also avoids the toy-like appearance of a train moving from a tangent directly into a sharp circular curve.

Prototype railroads also use superelevation, which banks the curved track, to minimize the effect of centrifugal force. Superelevated curves require easements. Along the easement length, the superelevation height gradually increases from zero at the tangent to its full value on the circular curve. Although superelevation on model railroads is unnecessary, it may be included for improved and more prototypical visual appearance. See **NMRA Data Sheet DS5.30 *Superelevation*** for guidance on its use.

3. Connecting Tangents with Curves and Easements

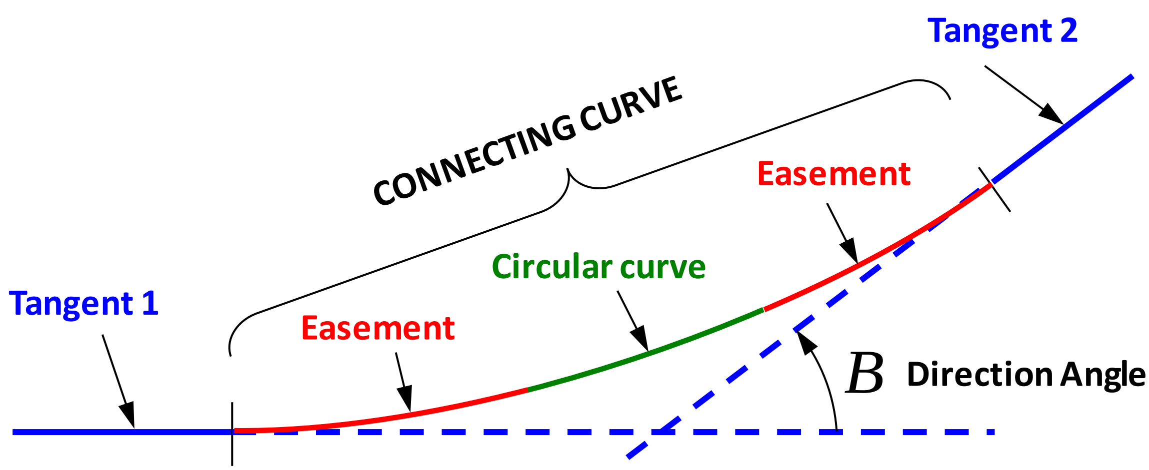

Figure 2: Direction Angle and Curve between Tangents (Drawing by Van S. Fehr)

Figure 2 illustrates the centerlines of two tangent tracks and the curved track required to connect them. The tangents (blue lines), arbitrarily labeled 1 and 2, are oriented by the direction angle between them, annotated by the letter *B*. The connecting curve is usually a combination of a circular curve (green line) of *constant* radius and an easement of *varying* radius on each end (red lines), as in Figure 1. The direction angle is important because it determines the type of easement that should be used on the connecting curve.

In the prototype, easements are normally one of two types. The first type is the cubic spiral easement and the second is the parabolic easement. When the direction angle is large enough,

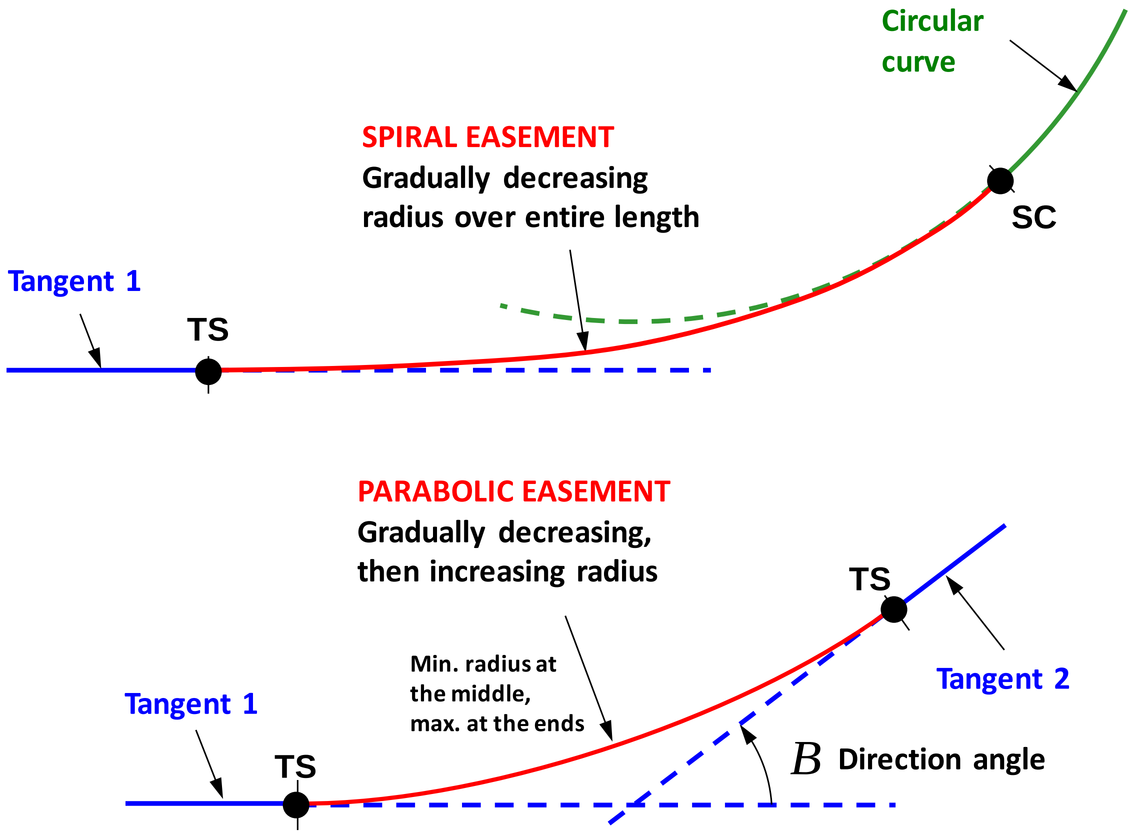

Figure 3: Easement Types (Drawing by Van S. Fehr)

a spiral easement connects each end of the circular curve to its adjacent tangent. If the direction angle is too small for a spiral easement, the connecting curve consists entirely of a single parabolic easement. How to make the choice is explained later in section **6. Choosing the Easement Type**.

Figure 3 illustrates these two easement types. The upper diagram is like that in Figure 2 but shows only the connection to Tangent 1. At the other end of the circular curve, the connection to Tangent 2 is essentially a mirror image. The point **TS** indicates where the tangent (blue line) connects to the spiral easement (red line), and the point **SC** indicates where the spiral easement connects to the circular curve (green line).

In the lower diagram, the radius of the parabolic easement varies continuously from a maximum at the ends to a minimum at the middle, so there is no green line indicating a constant radius circular curve. Because it replaces two spiral easements, a parabolic easement is about twice the length of a single spiral easement.

Guidance – Easements:

- *Wherever possible, model railroads should use easements to minimize coupler swing, avoid coupler and diaphragm binding, and achieve prototypical appearance.*

4. Laying Out Spiral Easements – the Bent Stick Method

In the early days of railroading, civil engineers used the complicated mathematics of the cubic *spiral* to prepare easement tables that helped surveyors stake out the easement centerline. Neither the precise math nor the tables for a cubic spiral are of practical use to model railroaders. What is useful is the cubic *spline*, which closely approximates the cubic spiral. Its math is far simpler. Some modelers use its math to lay out easement centerlines, sometimes preparing a template with one edge having the calculated shape of the easement. That edge is then used as a guide to draw the easement centerline on the sub-roadbed.

However, most modelers prefer using a thin, flexible stick of uniform cross-section to lay out the easement centerline. When gently flexed, the stick closely approximates the shape of a spiral easement and "does the math" automatically. This leads to the highly recommended, well-known, and widely used *Bent Stick Method* for laying out spiral easements.

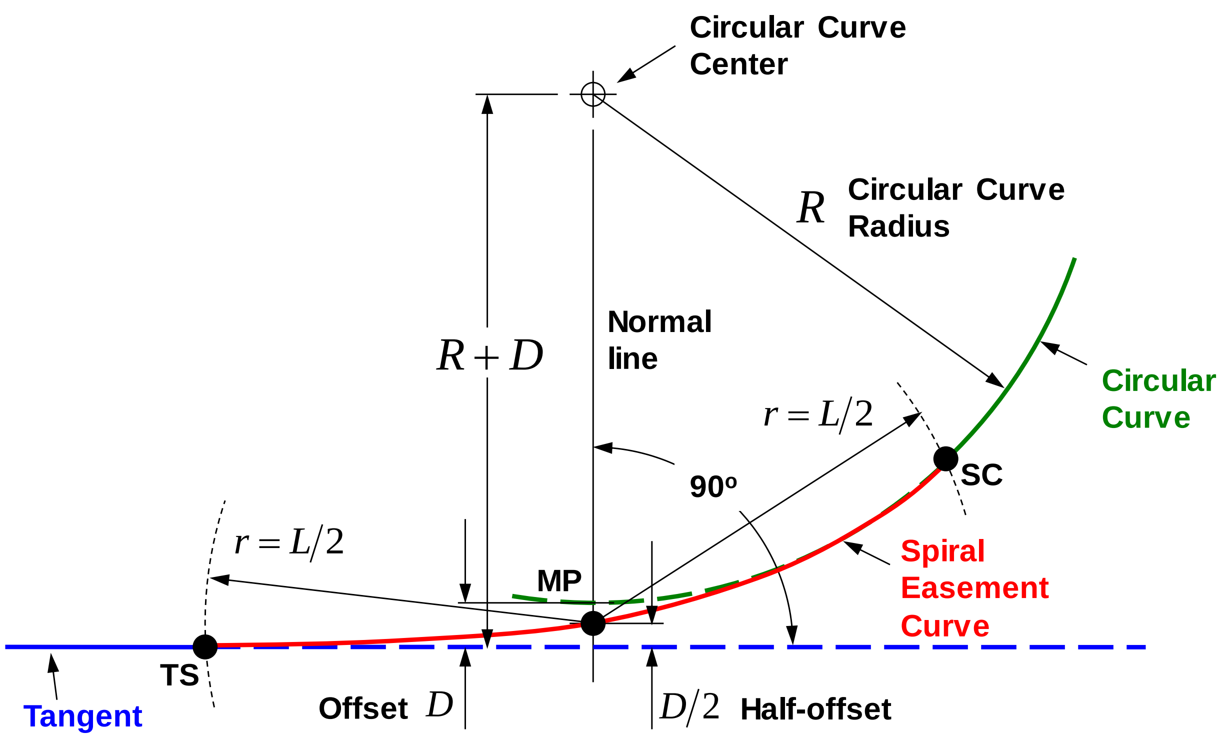

Figure 4: Spiral Easement Geometry (Drawing by Van S. Fehr)

Figure 4 illustrates the geometry of the Bent Stick Method. This method requires two key dimensions; the easement length L and the end-radius R, both selected by the modeler. The end-radius is also the radius of the circular curve (green line) that continues beyond the end of the spiral easement. The length of a prototype easement is often several hundred feet, far too long for a scale model railroad. Shorter easements are necessary for the model.

Guidance – Selecting Spiral Easement Length and End-Radius:

- *The minimum practical length of a spiral easement is the coupler-to-coupler length of the longest car or locomotive expected to traverse it. Exclude the tender for steam locos.*
- *One and a half times the minimum length is better, and twice the minimum length is best as a practical maximum. Longer easements are acceptable, space permitting.*
- *The end radius should not be less than the minimum radius recommended by **NMRA Recommended Practice RP-11 Curvature and Rolling Stock**.*

Proper use of the Bent Stick Method does require a few simple calculations. One important characteristic of a spiral easement is its aspect ratio:

$$\textbf{Aspect Ratio:}\ \ p = \frac{L}{R}$$

The aspect ratio is a useful geometric parameter, independent of model scale, and an indicator of the easement's prototypical appearance. Appearance is admittedly subjective. Prototype spiral easement aspect ratios are normally less than about 0.5 but can be larger on a model railroad. Model easements with aspect ratios between 1.0 and 1.5 are sometimes useful but not as prototypical looking. That's because the easement length is greater than the end radius, a highly unlikely occurrence in the prototype. Although model easements with aspect ratios greater than 1.5 are possible, they should be avoided because they are grossly non-prototypical. Modelers should make their own judgments but consider the guidance offered here.

Guidance – Easement Appearance (Aspect Ratio):

- ***Best** appearance: aspect ratio less than 0.5*
- ***Good** appearance: aspect ratio between 0.5 and 1.0.*
- ***Borderline** appearance: aspect ratio between 1.0 and 1.5.*
- ***Grossly un-prototypical** appearance: aspect ratio greater than 1.5.*

Another important parameter is the easement offset D, also calculated using p and L:

$$\textbf{Offset:}\ \ D = p\frac{L}{24}$$

The offset is the distance from the extended tangent line to a parallel line geometrically tangent to the extended circular curve, as Figure 4 illustrates.

Figure 5 shows how to apply the Bent Stick Method, using a photo of the flexed stick. It assumes the normal line, tangent, and circular curve have been laid out on the sub-roadbed, and the midpoint **MP** marked at one-half the offset above the extended tangent line. Swinging an arc of radius $L/2$ from the midpoint **MP** locates the point **TS** where the easement intersects

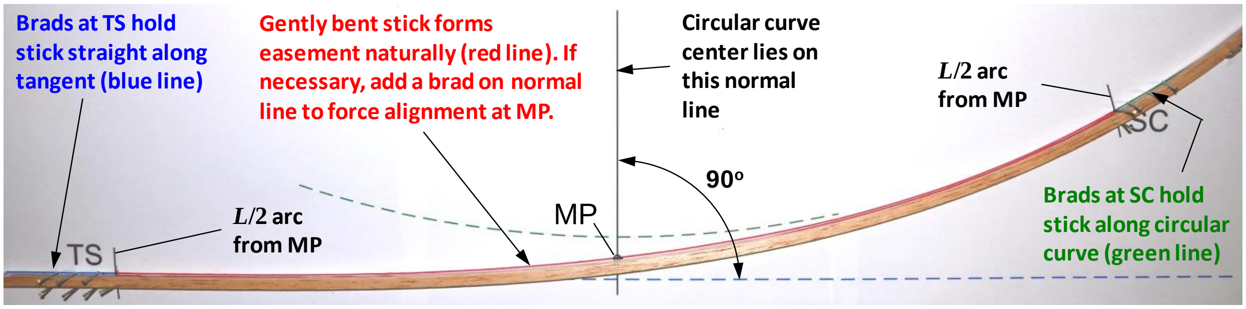

Figure 5: Bent Stick Method for Laying Out an Easement (Photo by Van S. Fehr)

the tangent and the point **SC** where it intersects the circular curve. It also shows how to align the stick that connects them. The brads at each end hold the stick in position while the easement is traced along the side of the gently flexed (bent) stick. If there is a noticeable gap between the stick and point **MP**, add a brad on the normal line to force the stick into alignment at point **MP**. Choosing a proper stick for the Bent Stick Method is important.

Guidance – Choosing the Stick:

- *The stick must have a rectangular or square cross-section whose dimensions do not change along its length.*
- *The stick should be about a foot longer than the easement length to ensure adequate overhang for positioning at the ends of the easement.*
- *The stick should be visually straight and return to its straight condition after being gently flexed to form the easement.*
- *A good stick choice for easements under 24 inches in length is a straight 3/16-inch square, 36-inch long balsa wood strip, available at many hobby shops, craft stores, and on-line retailers. Another good choice is a yard- or meter-length aluminum rule measuring about 1/16-inch thick. These rules are available at home improvement centers, like Home Depot, Lowes, and similar stores.*
- *For easements 24 to 84 inches long, a 1/2-inch or 3/4-inch wide strip ripped from 3/16-inch or 1/4-inch thick tempered hardboard (Masonite is one brand name), or something similar, is a good choice. Tempered hardboard is also commonly available at home improvement centers, typically in 4 feet by 8 feet sheets.*

For an innovative alternative to using brads to position the stick, see Bob Kingsnorth's tip ***"Build an adjustable guide to draw easement curves"*** **Model Railroader,** June 2010, p. 24**.**

For a given easement length, notice that as the specified end-radius becomes larger, the required offset gets smaller. Eventually it becomes so small that it is unnoticeable, as is the easement itself. In this case, no easement is necessary and is replaced entirely by the circular curve. What is unnoticeable is a judgment the modeler must make. It is also scale dependent. For example, if a one-quarter inch offset in HO scale is judged unnoticeable, then a one-half inch offset in O scale is probably unnoticeable, too.

Guidance – Offset:

- *Use a circular curve without a spiral easement when the offset is judged unnoticeable.*

5. Laying Out Parabolic Easements – The Bent Stick Method (Again)

While less common in the prototype and the model, the parabolic easement does have an important use when the direction angle is too small, and the spiral easement cannot be used (see section **6. Choosing the Easement Type**). Like the spline approximation for the spiral easement, the parabolic easement also has simple math, but none is required to lay one out using the Bent Stick Method. Once again, the bent stick does the math automatically.

However, there are a few simple differences. As mentioned earlier, the parabolic easement length should be roughly twice the length established for spiral easements. The longer lengths of the parabolic easement will limit the choice of stick (see ***Guidance – Choosing the Stick***).

Using the Bent Stick Method for laying out parabolic easements is simpler than using it for spiral easements. For simplicity, locate each end point **TS** at one-half the parabolic easement length from the intersection of the tangents (see Figure 3). Use brads (as in Figure 5) to force the stick into alignment along both tangents. Trace the easement along the edge of the stick, which now naturally takes the shape of a parabola.

6. Choosing the Easement Type

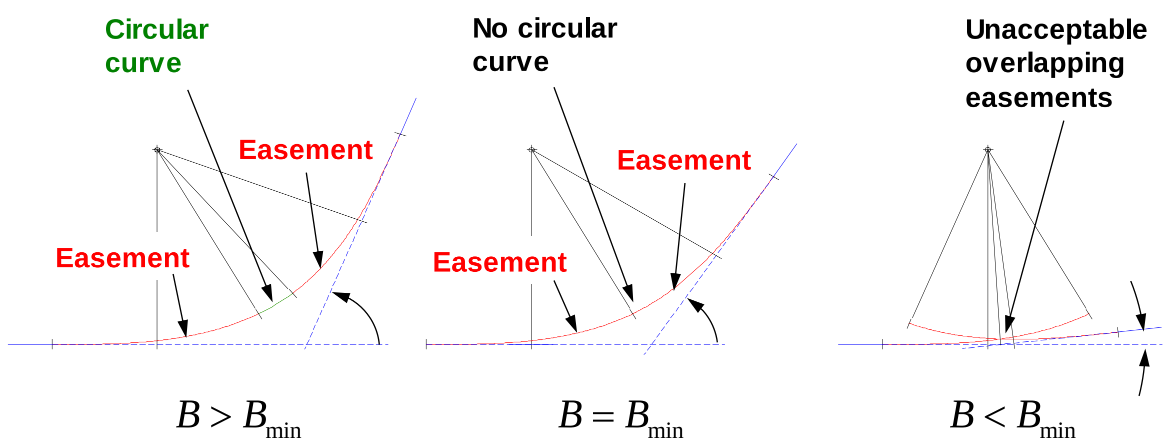

Figure 6: Easement Selection (Drawing by Van S. Fehr)

The type of easement to use is not simply a matter of personal preference. The proper choice depends on the value of the direction angle between the two tangents. Spiral easements on each end of a curve connecting two tangents have a minimum direction angle (measured in degrees) calculated using the aspect ratio defined earlier:

Minimum Direction Angle: $B_{MIN} = 57.3p$ degrees

When the direction angle is greater than the spiral easement minimum direction angle, there will be a circular curve between the two spiral easements, as shown at the left of Figure 6. When the direction angle equals the minimum direction angle, the easement entering the circular curve will end exactly where the departing easement begins, as shown in the middle. In this case, there is no circular curve at all. This is an acceptable, but rare, occurrence.

When the direction angle is less than the spiral easement minimum direction angle, the spiral easements will overlap as shown on the right, causing an unacceptable angular mismatch where they intersect. When this happens, ***spiral easements cannot be used***, and the connecting curve (see Figure 2) is replaced entirely by a ***parabolic easement***.

Guidance – Choosing the Easement Type:

- *Use a spiral easement if the direction angle is greater than the minimum direction angle.*
- *Use a parabolic easement if the direction angle is less than the minimum direction angle.*

7. Example – A Spiral Easement

A track plan has a right-angle turn (direction angle $B = 90$ degrees). The selected easement length is $L = 22.0$ inches, and the desired radius is $R = 30.0$ inches. Calculate the aspect ratio:

$$p = \frac{L}{R} = \frac{22}{30} = 0.733$$

This easement is in the ***good appearance*** range describe earlier. The minimum direction angle is

$$B_{MIN} = 57.3p = 57.3(0.733) = 42.0 \text{ degrees}$$

Because the required 90-degree direction angle is greater than the minimum direction angle, the spiral easement is the proper choice for each end of the connecting curve. The required offset is:

$$D = p\frac{L}{24} = (0.733)\frac{(22)}{(24)} = 0.67 \text{ inches}$$

8. Example – A Parabolic Easement

A track plan has a turn with a direction angle of 30 degrees, a desired radius of $R = 24.0$ inches, and a selected easement length $L = 22.0$ inches. The calculated aspect ratio and spiral easement minimum direction angle for this turn are:

$$p = \frac{L}{R} = \frac{22}{24} = 0.917$$

$$B_{MIN} = 57.3p = 57.3(0.917) = 52.5 \text{ degrees}$$

Because the spiral easement direction angle (30 degrees) is less than the minimum direction angle (52.5 degrees), a spiral easement will not work for this turn, requiring the use of a parabolic easement instead.

Because the parabolic easement replaces *two* spiral easements, choose a parabolic easement length of *twice* the chosen spiral easement length. In this case, 44.0 inches. No math is required, but for comparison, a circular curve connecting these two tangents would have a comparatively large radius of 85.0 inches.

9. Track Planning and Construction

While easements are recommended, they are not always practical because of space limitations. The modeler must make judgments for the specific planning problem at hand. Using easements on the mainline is always a good idea. In yards and sidings, there is usually insufficient room for them near the turnouts. To ensure adequate clearance, concentric yard tracks require centerline spacing wider than the parallel tangent tracks leading to/from them. Using easements can accomplish this, as described in "***How to plan concentric curves with easements,*" Model Railroad Planning 2011,** Kalmbach Publishing. pg. 62. Also see the **NMRA Recommended Practice RP-7 Track Centers and Obstacle Clearance** series for clearance guidance.

The legs of a reversing wye likely do not have enough room for easements, while the legs of a reversing loop often do. When limited space prevents use of an easement, modelers should be mindful of the curve radius, making sure the equipment expected to traverse it can do so without coupler binding, especially where the curve joins the tangent. Again, see **NMRA Recommended Practice RP-11 Curvature and Rolling Stock** for guidance.

The Bent Stick Method described in this Data Sheet was originally developed for use during construction. Today, many model railroaders use a Computer Aided Design (CAD) computer program to prepare detailed track plans. Most CAD programs can replicate the Bent Stick Method using their built-in "spline tool" or similar functionality. Some CAD programs designed specifically for model railroad track planning, such as *3rdPlanIt* and *CadRail*, include an "easement tool" for this purpose.

For easement construction, the final test is for visual smoothness. Some inaccuracy will always be present in construction, no matter how much care is taken. Once drawn on the sub-roadbed, sight along the easement, and make sure there are no visible reverse curves or angular kinks along its length, or at its endpoints. If there are none, installation of the roadbed (ballast strip) and track follows. A final sight check for smoothness completes the process. If the easement track is visually smooth, it will serve its purpose and operate reliably.

10. Use Easements

Model railroaders have understood and used easements from "day one." The third issue of ***The Model Railroader*** (March 1934) included an article titled "*Laying Out a Parabolic Curve.*" A little over six years later, a cartoon in the June 1940 issue (Figure 7) humorously admonished poor Van for his failure to use them. "Use easements" is still good advice today.

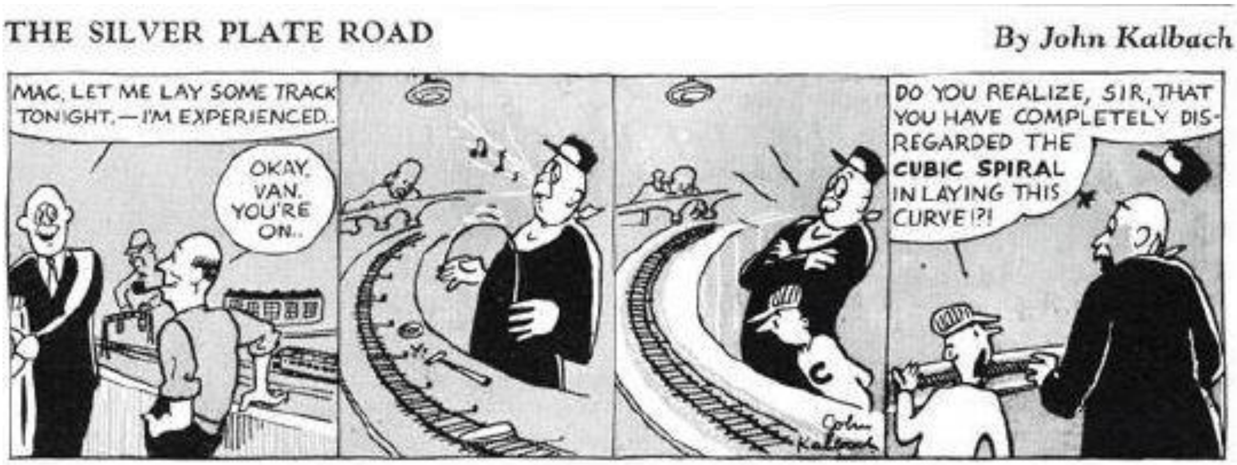

Figure 7: Use Easements! (Cartoon courtesy of *Modeler Railroader*)